

Saturation of Warming by the Greenhouse Gas CO₂

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At current atmospheric concentrations C of carbon dioxide (CO₂), there is less and less increase of the temperature T as more and more of the greenhouse gas CO₂ is added. This law of diminishing returns is called *saturation* in radiation-transfer physics.

We will quote concentrations C in units of CO₂ molecules per million air molecules, or ppm. It is useful to define a dimensionless *relative concentration* of CO₂ by

$$c = \frac{C}{\delta C} \tag{1}$$

where the *saturation concentration*,

$$\delta C = 0.391 \text{ ppm}, \tag{2}$$

which was discussed in reference [1]. For concentrations $C \ll \delta C$, the CO₂-induced greenhouse warming is very small but it is proportional to C . For $C \gg \delta C$, the warming is proportional to $\log_2(C/\delta C)$, where $\log_2$ is the base-2 logarithm function, for example $\log_2(1) = 0$, $\log_2(2) = 1$, $\log_2(4) = 2$, *etc.* It is also useful to define a dimensionless *relative warming*

$$t = \frac{T}{\delta T}. \tag{3}$$

Here we will take *temperature sensitivity* to be [1]

$$\delta T = 0.54 \text{ C}, \tag{4}$$

As discussed in reference [1], saturation can be summarized by a simple formula that gives the relative warming t in terms of the relative concentration c of CO₂,

$$t = \log_2(1 + c). \tag{5}$$

The inverse of (5) is

$$c = 2^t - 1. \tag{6}$$

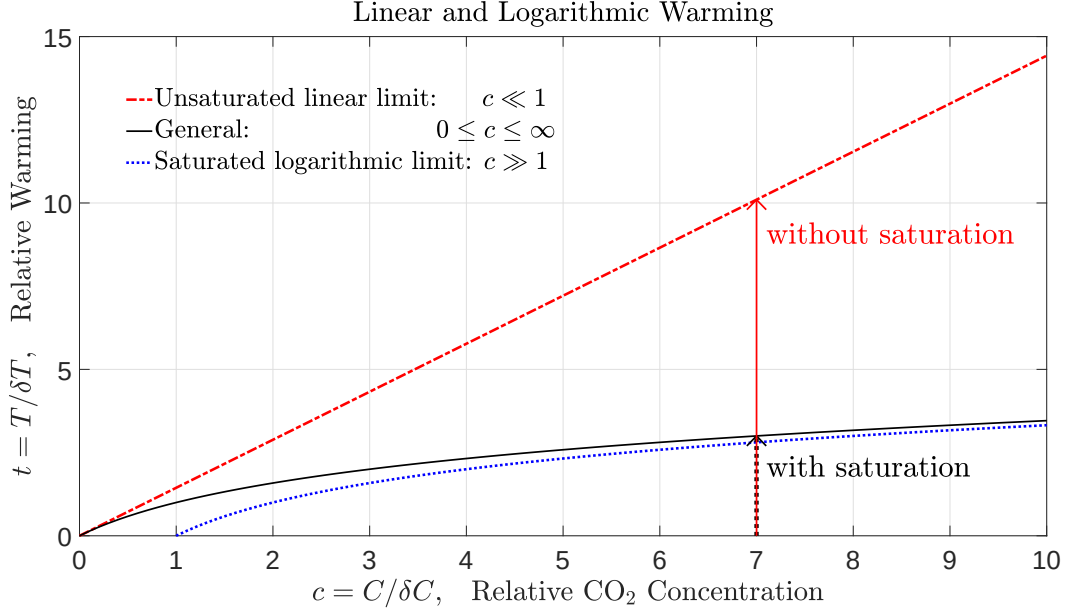


Figure 1: The relative warming t of (3) versus the relative concentration c of (1), as given by: (1) the high-concentration, saturated approximation (7); the generally correct formula (5); (3) the low-concentration, unsaturated approximation (9). As shown in (8), in the current year, 2026, atmospheric concentrations of CO_2 , $C = 430$ ppm, correspond to a relative concentration (1) of $c = 1100$. So current warmings are very well described by the high-concentration, logarithmic limit (7).

For the limit of very large relative concentrations, $c \gg 1$, we can approximate the general warming formula (5) by

$$t \approx \log_2(c) \quad \text{for } c \gg 1. \quad (7)$$

In the year 2026, the CO_2 concentration C and its relative value c of (1) are approximately

$$C = 430 \text{ ppm} \quad \text{and} \quad c = 1100. \quad (8)$$

Then the exact value of (5) is $t = \log_2(1 + c) = \log_2(1101) = 10.1046$ and the approximate value (7) is $t = \log_2(c) = \log_2(1100) = 10.1033$. Therefore, at the present time, the year 2026, (7) is a very good approximation to (5). The logarithmic formula (7) fails completely for $c \leq 1$ since it gives negative relative warming, $t < 0$, and it implies an infinite negative warming as the CO_2 concentration drops to zero. As $c \rightarrow 0$ in (7), $t = \log_2(c) \rightarrow -\infty$.

To find a limiting expression for (5) for $c \ll 1$, analogous to the approximation (7) for $c \gg 1$, we write (5) as

$$\begin{aligned} t &= \log_2(1 + c) = \frac{1}{\ln 2} \ln(1 + c) \\ &= \frac{1}{\ln 2} \left(c - \frac{c^2}{2} + \frac{c^3}{3} - \frac{c^4}{4} + \cdots \right) \\ &\approx \frac{c}{\ln 2} = 1.4427c \quad \text{for } c \ll 1. \end{aligned} \quad (9)$$

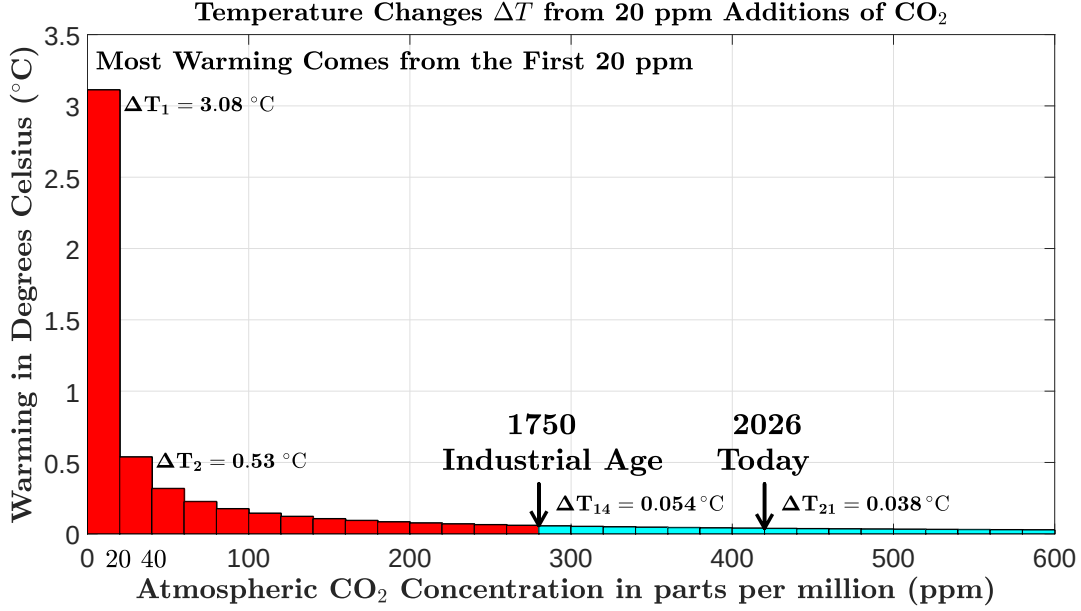


Figure 2: The warmings, ΔT_n from successive additions of $\Delta C = 20$ ppm increments of CO_2 to the atmosphere. The concentration after the first addition is $C_1 = 20$ ppm, after the second addition, $C_2 = 40$ ppm, etc. Over half of the total warming by the year 2026, $T_n = 5.44$ C, when the concentration was $C_n = 420$ ppm, was due to the first 20 ppm addition of CO_2 , which caused a warming of $\Delta T_1 = 3.08^\circ\text{C}$. Representative numerical values are shown in Table 1.

Here $\ln(x) = \log_e(x)$ is the natural, base- e logarithm of the variable x . In writing the second line of (9) we used the power series expansion

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad (10)$$

and we noted the identity

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}. \quad (11)$$

For example, if $c = 0.01$ the exact warming from (5) is $t = \log_2(1.001) = 1.44 \times 10^{-3}$ and the approximation (9) gives same number.

The general formula (5) for the relative warming t caused by the relative CO_2 concentration c is plotted in Fig. 1, along with the large-concentration limit (7) and small-concentration limit (9). In the year 2026, when the relative concentration is approximately $c = 1100$, according to (8), using the small-concentration warming formula (9), which does not account for saturation, instead of the correct general formula (5) that does, would exaggerate the predicted warming by a factor

$$\begin{aligned} f &= \frac{c}{\ln 2 \log_2(c)} \\ &= \frac{c}{\ln(c)} = 157. \end{aligned} \quad (12)$$

n	C_n (ppm)	ΔT_n (° C)	T_n (°C)
0	0	0	0
1	20	3.08	3.08
2	40	0.53	3.61
3	60	0.31	3.93
...	...	...	...
14	280	0.058	5.12
15	300	0.054	5.18
...	...	...	...
21	420	0.038	5.44
22	440	0.036	5.47
...	...	...	...
29	580	0.027	5.69
30	600	0.026	5.72

Table 1: The number n of concentration increments, $\Delta C = 20$ ppm, added to the atmosphere is shown in the first column. The second column shows the cumulative concentration C_n of (14). The third column shows the incremental warming ΔT_n of (19), caused by adding the n th increment, $\Delta C = 20$ ppm, of CO₂. The fourth column shows the cumulative warming T_n of (18). Fig. 2 is a graphical display of the numbers in this table.

To write the second line of (12) we used (11).

Imagine that we increase the concentration of CO₂ in equal 20 ppm increments,

$$\Delta C = 20 \text{ ppm} \quad (13)$$

After n increments, starting from $C_0 = 0$, the concentration would be

$$C_n = n\Delta C = n \text{ 20 ppm} \quad \text{for } n = 0, 1, 2, 3, \dots \quad (14)$$

The relative concentrations would be

$$c_n = rn \quad \text{for } n = 0, 1, 2, 3, \dots \quad (15)$$

where the *saturation ratio* is

$$r = \frac{\Delta C}{\delta C} = 51.2. \quad (16)$$

Substituting (15) into (5) we see that adding n increments ΔC of (13) would increase the relative temperature by

$$t_n = \log_2(1 + rn), \quad (17)$$

From (17) and (3) we see that the temperature increase in degrees would be

$$T_n = \delta T t_n = \delta T \log_2(1 + rn), \quad (18)$$

Using (18) we see that the temperature increment caused by adding the n th increment (13) to the atmosphere would be

$$\begin{aligned}
\Delta T_n &= T_n - T_{n-1} \\
&= \delta T \log_2 \left(\frac{1 + nr}{1 + [n-1]r} \right) \\
&\approx \delta T \log_2 \left(\frac{n}{n-1} \right) = \frac{\delta T}{\ln(2)} \ln \left(\frac{n}{n-1} \right) = -\frac{\delta T}{\ln(2)} \ln \left(1 - \frac{1}{n} \right) \\
&= \frac{\delta T}{\ln(2)} \left(\frac{1}{n} + \frac{1}{2n^2} + \frac{1}{3n^3} + \frac{1}{4n^4} + \cdots \right) \\
&\approx \frac{\delta T}{n \ln(2)} \quad \text{for } n \gg 1.
\end{aligned} \tag{19}$$

The second line of (19) is exact for $n = 1, 2, 3, 4, \dots$. The third line of (19) is a very good approximation to the second for $n = 2, 3, 4, \dots$, because $r \gg 1$ in accordance with (16). In the fourth line we used the power series expansion (10).

Fig. 2 shows the incremental warmings ΔT_n of (19) caused by the n th addition of $\Delta C = 20$ ppm to the atmosphere. Also shown are the cumulative CO₂ concentrations C_n of (14) after the n th addition, and the cumulative warming temperatures T_n of (18). More than half of the warming in the year 2026, when the CO₂ concentration is approximately $C = 430$ ppm, has been caused by the first 20 ppm increase in the concentration of CO₂ from a vanishing initial concentration $C_0 = 0$. Current additions of CO₂ have very little effect because of the heavy saturation. Table 1 gives numerical values of some of the data plotted in Fig. 2.

References

- [1] Saturation Graphics,
<https://co2coalition.org/publications/saturation-graphics/>